

第164回

附属社会創造数学センター主催
学術変革領域研究(A)「マルチモーダルECM」共催

HMMCセミナー

Date : 2025年9月18日(木) 16:30~18:00

Speaker : 長藤 かおり (Kaori Nagatou)
Karlsruhe Institute of Technology, Germany

Style: 対面とのオンラインのハイブリット開催
オンラインのみ要事前登録

オンラインの参加申し込みは下記URLからお願いします。

<https://zoom.us/meeting/register/LSmc3K23TBm4jUyRi0nUiQ>

Place : 北海道大学 電子科学研究所
中央キャンパス総合研究棟2号館5階 講義室

Title : Non-selfadjoint spectral problems related to self-similar
blowup in nonlinear wave equations

Non-selfadjoint spectral problems related to self-similar blowup in nonlinear wave equations

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We consider the wave equation with a power nonlinearity

$$u_{tt} - \Delta u = u^p,$$

with initial profiles $u(x, 0)$ and $u_t(x, 0)$, $x \in \mathbb{R}^3$, $t \geq 0$, and $p > 1$ an odd integer. In order to investigate the blowup dynamics we look for radial self-similar blowup solutions of the form

$$u(x, t) = (T - t)^{-\frac{2}{p-1}} U\left(\frac{|x|}{T - t}\right), \quad T > 0,$$

with a smooth, radial profile U . In particular, we are interested in stability properties of such solutions. This gives rise to analyzing the spectrum of the linearized operator, i.e. to the non-selfadjoint eigenvalue problem:

$$\mathcal{L}\mathbf{u} = \lambda\mathbf{u},$$

where $D(\mathcal{L}) \subset H_{\text{rad}}^2(B^3) \times H_{\text{rad}}^1(B^3)$, $H_{\text{rad}}^k(B^3) := \{\mathbf{u} \in H^k(B^3) : \mathbf{u} \text{ is radial}\}$,

$$\mathcal{L} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} := \begin{pmatrix} -\rho u_1'(\rho) - \alpha u_1(\rho) + u_2(\rho) \\ u_1''(\rho) + \frac{2}{\rho} u_1'(\rho) - \rho u_2'(\rho) - (\alpha + 1)u_2(\rho) + V(\rho)u_1(\rho) \end{pmatrix},$$

$B^3 = \{x \in \mathbb{R}^3 : |x| \leq 1\}$, $\rho = \frac{|x|}{T-t}$, $V(\rho) = pU(\rho)^{p-1}$ and $\alpha = \frac{2}{p-1}$. We are interested in excluding eigenvalues of $\mathcal{L}$ in the crucial parts of the right complex half plane, which is ongoing work together with B. Schörkhuber, Y. Watanabe, M. Plum and M.T. Nakao. We first provide a compact set $R \subset \mathbb{C}$ such that no eigenvalues can exist in the right half-plane outside this set. Furthermore, we derive estimates ensuring that small discs with explicitly computable radii and carefully chosen centers are free of eigenvalues. Covering the set R with such discs gives, in principle, the desired non-existence of eigenvalues, implying (linear) stability.